DEFORMATION THEORY OF QUANTUM FIELDS

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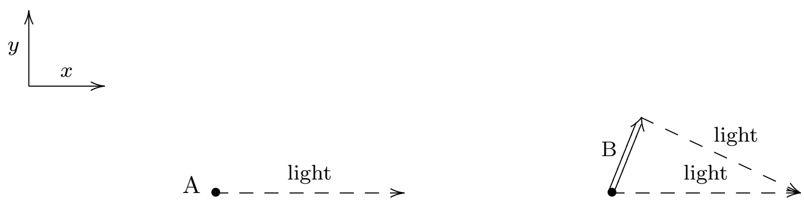
Abstract. In this paper, from the standpoint of quantum field theory, we shall study an analogous theory with the deformation theory of elliptic curves over $\mathbb{Q}$ obtained in [Mo2].

1. Special relativity

Let A be an object on the ground and B be an object moving at the constant speed V. Then, by the theory of special relativity, A sees that time goes more slowly around B than B itself feels. It is assumed that this phenomena, regardless of the presence or absence of A and B, results from the Lorentz transformation between the stationary inertial frame of reference with A and the inertial frame of reference moving with B. In this paper, however, we will regard that time of flow is adjusted by exchanging the virtual propagators between A and B. For example, in the case $V = 0$, A and B share the same time and, in the case $V = c$ (the speed of light), A sees that time stops around B.

Let us consider the situation above. A person on the ground sees that both of the light from the stationary object A and the light from the object moving at the speed V move at the speed c and that the formula of vectors is not established. In [Mo3], we set up a mathematically virtual space where the formula of vectors is concluded.



A person on the ground can see only the x-axis and the formula of vectors is concluded in this mathematically virtual world (x-y plane) which is invisible to the person. Even if the object B moves at the more speed, the person on the ground will see that the light from the object B always moves at the speed c. Thus, it is no wonder that, if the object B moves faster, the more momentum is annihilated to the direction of y-axis and this quantity does not contribute to the

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observation at the object A. Therefore, we can think of the virtual propagators adjusting time of flow as the virtual propagators adjusting momentum. That is to say, these virtual propagators send the distortion of space-time from the object A and these propagators will play an key role in the deformation theory of quantum fields.

2. Deformation theory of quantum fields

2.1. **Modularity and wave nature.** We denoted in [Mo3] that, in number theory, there exists each different world for each prime number and similarly that, in physics, there exists each different world for each inertial frame of reference. Furthermore, in [Mo4], when the set of all momentum is quantized, we obtained analogous results in physics with results obtained in number theory. The reason why we studied such analogies is that we think that the modularity in number theory is similar to the wave nature in physics.

2.2. **Modular forms.** Before we study the deformation theory of quantum fields, let us review the deformation theory of elliptic curves over $\mathbb{Q}$ ([Mo2]). For an elliptic curve E over $\mathbb{Q}$, its holomorphic differential form $f(z)$ (called modular form) is defined over the upper half space $\mathbb{H}$ and is invariant under the modular group. As significant nature, the Fourier expansion of this modular form is given by

$$f(z) = \sum_{n=1}^{\infty} a_n(f)q^n \quad (a_n(f) \in \mathbb{C},\ q = e^{2\pi iz})$$

and we have the equality $a_p = 1 + p - \sharp E(\mathbb{Z}/p\mathbb{Z})$ for almost all prime numbers p which contains information about the elliptic curve modulo p. It should be noted that, if n is a composite number, the coefficients a_n is determined by $\{a_p\}_{p:\text{prime}}$. On the other hand, each coefficient cannot be selfish behavior since it is determined by the global elliptic curve E over $\mathbb{Q}$. In fact, it is known that a modular form is determined if almost all the Fourier coefficients are determined. This property is called modularity in number theory. In [Mo2], we studied how the rational points on elliptic curves behave when each Fourier coefficient a_p is deformed continuously.

2.3. **Deformation theory of quantum field operators.** The coefficients of the momentum expansion of a quantized wave function cannot be also selfish behavior by the wave nature. The aim of this paper is to deform these coefficients continuously and study their behaviors. Concretely, consider the quantum field operator $\phi(x,t)$ of a quantized wave function and write its momentum expansion by the creation-annihilation operators $\{a_{\mathbf{p}}\}$ and $\{a_{\mathbf{p}}^{\dagger}\}$. Then, deform these coefficients continuously. This deformed function no longer has any regularity derived from the wave nature.

2.4. **Mathematical interpretations.** There exists each different world for each prime number in number theory and similarly there exists each different world for each inertial frame of reference in physics. Based on this thing, we focus on one momentum **p** and we consider a function $\phi'(x,t)$ which is obtained by deforming the coefficients of $a_{\mathbf{p}}$ and $a_{\mathbf{p}}^{\dagger}$ continuously. Let us regard that the propagators in the previous section are emitted to the virtual wave $\phi'(x,t)$ from the stationary object A. Then, these propagators feel the state change of the world for the inertial frame of reference with the momentum **p**. Conversely, we can say that a new space-time without regularity of the wave nature is created by these propagators.

3. Some comments

3.1. **As mathematical methods.** It is difficult to treat the quantum field operators strictly but if we consider a new space-time without regularity of the wave nature, we expect that we can use the topological methods in studying quantum field theory. This is an analogy with the methods by which we research the topological aspects of the arithmetic elliptic curves over $\mathbb{Q}$. Furthermore, in Feynman diagrams, the virtual propagators in Section 1 will be useful to visualizing the changes of space-time.

3.2. **Application to string theory.** In string theory, it is believed that the dimension of space-time is 10 or 26 and that the reason why we regard this world as the 4 dimensional world is that the superfluous dimensions are rounded small by the compactification. Assume that a string turns round the inner space with the wave nature. By deforming the world of each momentum **p** continuously, let us consider a new space-time without regularity of the wave nature. Then, in that new space-time, the future of the string will be observed as straddling the inside and outside of the inner space.

3.3. **Inner structure of a string.** By string theory, it is believed that all substances consist of not points but strings. Then, it is interesting to consider the inner structure of a string. Let us interpret this by using the new space-time introduced above. The difference between a string and its deformed string will be a point as a limit when we narrow two strings again. It is natural to think of this point as the inner structure of a string. We cannot observe this limit point in our space-time. We can regard, however, that, only after we set up the new space-time introduced above, the virtual propagators sending the distortion of space-time will capture this inner structure. Alternatively, we can think that these propagators make this new space-time and as a result, this inner structure is created.

References

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