$SU(3)$ GRAND UNIFIED THEORY

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Abstract. The current mainstream model of the grand unified theory is based on the $SU(5)$ gauge group and assumes that $SU(3) \times SU(2) \times U(1)_Y$ gauge symmetry remains after the symmetry breaking. According to this theory, there exist 24 gauge fields with equal forces at the beginning of the universe. Indeed, one coupling constant unifies all the interactions, but the number of parameters is as many as 24, which is somewhat unsatisfactory for a unified theory. In this paper, we assume that the universe begins with $U(1)$ gauge symmetry and construct the grand unified theory as a $SU(3)$ gauge symmetry model.

1. $U(1)$ gauge symmetry

At the beginning of the universe, photons appear and the whole universe has $U(1)$ gauge group action. At this stage, only the charged electron e^- is affected by the electromagnetic interaction. Next, quarks constituting hadrons emerge and they have $SU(2)$ and $SU(3)$ gauge group actions as those producing interactions between the quarks. We should consider that the $SU(2)$ gauge symmetry, which is an interaction between two bodies, arose before the $SU(3)$ gauge symmetry, which is an interaction between three bodies. Assuming the existence of a unified theory, $SU(2)$ contains $U(1)$ as a subgroup at this stage.

2. $SU(2)$ gauge symmetry

As the maximal torus of the $SU(2)$ gauge group with quarks (leptons) as doublets, the embedding of the $U(1)$ gauge group is given by

$$U(1) \to SU(2) : e^{-i\theta} \mapsto \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix}.$$

With this embedding, quarks (leptons) become charged and exchange their charges due to the phase difference between the diagonal components of the matrix. In this primitive situation, it is reasonable to assume that the charge exchanged is the smallest unit of charge q, so that the phase difference 2θ corresponds to q. For example, the embedding of the electromagnetic interaction into the $SU(2)$

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gauge group with electron e^- and electron neutrino ν_e as the doublet is, by using the isomorphism $U(1) \simeq U(1) : e^{-iq\theta_W} \mapsto e^{-\frac{1}{2}iq\theta_W}$, obtained as follows

$$U(1) \to SU(2) : e^{-iq\theta_W} \mapsto \begin{pmatrix} e^{-i\frac{q}{2}\theta_W} & 0 \\ 0 & e^{i\frac{q}{2}\theta_W} \end{pmatrix} \xrightarrow{\text{twist}} e^{-i\frac{q}{2}\theta_W} \begin{pmatrix} e^{-i\frac{q}{2}\theta_W} & 0 \\ 0 & e^{i\frac{q}{2}\theta_W} \end{pmatrix}.$$

On the other hand, since the phase difference 2θ corresponds to q, the confinement of quarks (leptons) with charge $\pm\frac{1}{2}$ is also possible but is not realized due to the $SU(2)$ gauge symmetry breaking. The $SU(2)$ gauge symmetry requires the weak boson $W^\pm$ in order for quarks (leptons) with different charges to exchange their charges. The number of gauge fields of $SU(2)$ gauge group is 3 and the remaining one is the uncharged Z boson. If the $SU(3)$ gauge symmetry, a three-body interaction, arises from these components of the two-body interaction, then, as in the previous section, $SU(3)$ contains $SU(2)$ as a subgroup at this stage.

3. $SU(3)$ gauge symmetry

Let u, d be quarks with different charges obtained by $SU(2)$ gauge symmetry and let u have charge q higher than d accoring to the phase difference 2θ. In this case, we consider (d, u, d) as the triplet of the $SU(3)$ gauge group. The embedding of the $SU(2)$ gauge group into the $SU(3)$ gauge group is, by using Lie algebra, obtained as follows

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \mapsto \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \mapsto \begin{pmatrix} 0 & -i & 0 \\ i & 0 & i \\ 0 & -i & 0 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \mapsto \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Then, the embedding of the $U(1)$ gauge group into the $SU(3)$ gauge group is given by

$$U(1) \to SU(3) : e^{-i\theta} \mapsto \begin{pmatrix} e^{-i\theta} & 0 & 0 \\ 0 & e^{2i\theta} & 0 \\ 0 & 0 & e^{-i\theta} \end{pmatrix}.$$

Here, the phase difference 3θ corresponds to q and if the sum of charges of the triplet (d, u, d) is normalized to 0, the charges of each u, d can be determined as $\frac{2}{3}, -\frac{1}{3}$. Then, the embedding of electromagnetic interaction into the $SU(3)$ gauge group is, by using the isomorphism $U(1) \simeq U(1) : e^{-iq\theta_S} \mapsto e^{-\frac{1}{3}iq\theta_S}$, obtained as follows

$$U(1) \to SU(3) : e^{-iq\theta_S} \mapsto \begin{pmatrix} e^{-\frac{1}{3}iq\theta_S} & 0 & 0 \\ 0 & e^{\frac{2}{3}iq\theta_S} & 0 \\ 0 & 0 & e^{-\frac{1}{3}iq\theta_S} \end{pmatrix}.$$

In this case, the quark has fractional charge, which leads to the confinement of the quark. Furthermore, in this closed system, the quark must be color-charged due to Pauli's exclusion law for Fermi particles. In this way, the $SU(3)$ gauge symmetry arises and the eight different gluon bosons g are required.

4. SYMMETRY BREAKING

The breaking of $SU(2)$ gauge symmetry by the Higgs mechanism leaves $U(1)$ and $SU(3)$ gauge symmetries and the world as we know it today is created. On the other hand, if there is no neutrino ν_e which is a lepton, the only difference between quarks is the electron e^-, which cannot break the $SU(2)$ gauge symmetry. Regarding the type of quarks (leptons), it is known that the existence of more than three generations of quarks (leptons) is required to break the CP symmetry.

Having constructed the models of the $U(1), SU(2)$ and $SU(3)$ gauge symmetry, let us consider the gauge theory of the gravity. Assuming that the gauge group of the interaction of the gravity is the Lorentz group $O(1,3)$, it does not contain a subgroup $U(1)$, which was the smallest gauge group, so the $U(1)$ gauge symmetry and $O(1,3)$ gauge symmetry are independent though the intersection of both groups is the trivial finite group. Considering how the $SU(3)$ gauge symmetry is constructed in this paper and the energy density of the universe at that procedure, if there exists a Super Unified theory, the gauge group containing both $U(1)$ and $O(1,3)$ was broken long before the $SU(3)$ gauge symmetry arose.